# **Doing and Talking Mathematics:**
# A Teacher's Guide to Meaning-Making with English Learners

**Authors**

Rita MacDonald, Wisconsin Center for Education Research
H. Gary Cook, Wisconsin Center for Education Research
Sarah Lord, Doctoral student, School of Education, University of Wisconsin-Madison
Nora G. Ramirez, Mathematics Education Consultant, Tempe, AZ

**Consultants**

Judit Moschkovich, Professor of Mathematics Education, University of California-Santa Cruz.



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## Introduction

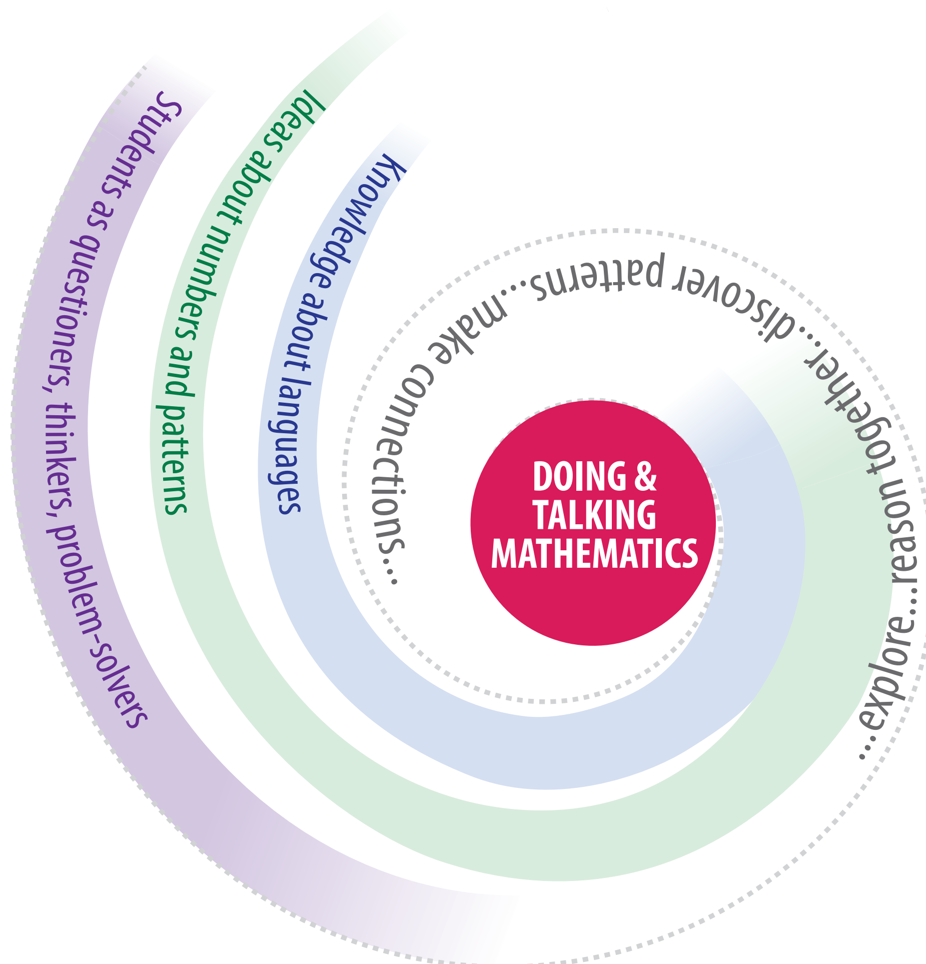
Across the United States, educators are changing the way they teach mathematics. These changes affect all students, including English Learners. New math standards have led to an increased focus on meaning-making. But, what is meaning-making in mathematics?

*Mathematics is not a list of disconnected topics, tricks, or mnemonics; it is a coherent body of knowledge made up of interconnected concepts.*

*Key Shifts in Mathematics, Introduction, Common Core State Standards for Mathematics*
www.corestandards.org/other-resources/key-shifts-in-mathematics/

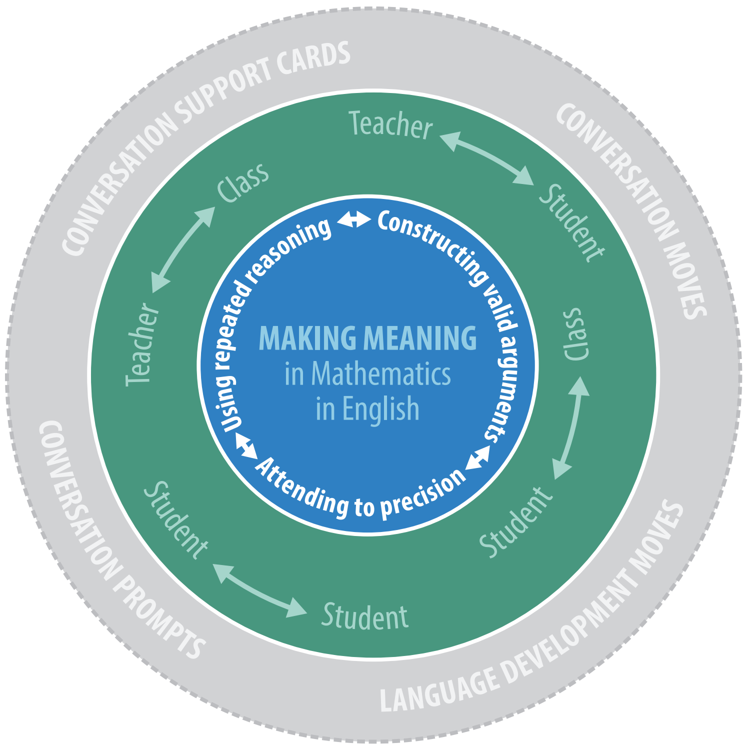
And how are those connections made? What produces the coherence?

*"Communication works together with reflection to produce new relationships and connections. Students who reflect on what they do and communicate with others about it are in the best position to build useful connections in mathematics." (Hiebert et al, 1997, p. 6)*




More than ever before, mathematics instruction is focused on helping students reason together, guiding students to discover patterns, make sense of mathematical concepts, and make connections between prior knowledge and new learning. Starting with problems that embody important mathematical ideas, students will reason through to the abstract concepts that make mathematics—and the world of numbers and relationships—make sense. For English Learners, this mathematical sense-making is accompanied by a simultaneous growth in their ability to make meaning in English. The interplay between meaning-making in mathematics and meaning-making in English strengthens students' ability in both domains.

Like all students, English Learners come to school with many experiences of numbers, shapes, patterns and relationships. These form the foundation upon which they will build knowledge. And like all students, English Learners are learning how to frame their ideas in increasingly precise language. With their classmates, they will explore concepts, explain their thinking, critique and justify claims, and uncover the mathematics involved while they learn through collaboration and experience how to communicate their growing understanding of mathematics in English.




This guide was developed to help mathematics teachers tap into the knowledge and experience that English Learners bring to their mathematics classrooms, and to help all students develop the language to be successful collaborators and meaning-makers in mathematics. The figure at the right shows a model of mathematics teaching and learning in which teachers support students as active and collaborative meaning-makers.

We've centered our work around three important practices in mathematics: **constructing viable arguments and critiquing the reasoning of others, attending to precision,** and **expressing regularity in repeated reasoning.** You can read more about the mathematical practices and how they interact in meaning-making later in this guide.

Just as the iterative cycling among these practices supports meaning-making, the interaction of the **Teacher Moves** and **Student Moves** outlined in this guide help build productive conversations in which teachers facilitate students' reasoning while pressing for clarity and deep thinking. This process helps students build ideas together. We've developed two specific teacher tools: **Productive Conversation Prompts** that help facilitate collaborative meaning-making and **Language Development Moves** to encourage students' language development. We've provided two specific tools for students to use, as well. **Productive Conversation Moves** show how students at three levels of English proficiency can use language to engage in seven key interactions as they develop and build ideas together. The **Conversation Support Card** is a resource that students can use in their small working groups, to help them ask and answer questions with one another as they work together to build understanding through the cyclic process of developing and critiquing arguments to build increasingly precise explanations in mathematics.

Constructing an environment in which students operate as meaning-makers requires careful preparation. Elements of preparing to teach for meaning are discussed in the following section. You will also find, in this section, an extensive set of **Classroom Supports** developed specifically to support student engagement in these mathematical practices. Finally, a **Lesson Scenario** will give you an overview of how these elements are integrated into a mathematics classroom.

# Teaching for Meaning

## Considering Important Mathematical Practices

Writers of the Common Core State Standards in Mathematics (CCSSI 2010) (which have either been adopted by states or have guided the revision of state mathematics standards), have emphasized the importance of the eight Mathematical Practices (MPs) in identifying important practices that underlie and support meaning-making in mathematics. They articulate what it means for students to *do mathematics.*

The MPs operate across all grade levels and all disciplinary core ideas. Three of these, shown in bold type, are language-rich practices that provide opportunities for meaning-making in both mathematics and English. We will focus on these three throughout this guide. While these are not the only practices that provide opportunities for meaning-making in both math and language, we are choosing to focus on these because they provide particularly rich opportunities—and because the guidance we give with regard to these three practices will apply to the other practices as well. The central role of these three MPs in mathematical sense-making is shown below.

**The Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. **Construct viable arguments and critique the reasoning of others**
4. Model with mathematics
5. Use appropriate tools strategically
6. **Attend to precision**
7. Look for and make use of structure
8. **Look for and express regularity in repeated reasoning**

The practice of **looking for and expressing regularities** can be a vehicle through which students build concepts and make sense of their procedural activities… look for patterns, consider generalities and limitations, and make connections across past and present bouts of reasoning. – p. 106

"Teaching mathematics is an incredibly complex task. Teachers must help students learn mathematical content while also guiding them in what it means to do mathematics. The eight Standards for Mathematical Practice in the new Common Core Standards for School Mathematics provide a vision of doing mathematics."
—*Connecting the NCTM Process Standards and the CCSSM Practices.* Koestler, Felton, Bieda, & Otten (2013), p. ix.

**Learning how to argue** whether an idea or claim is true or false in a mathematically valid way is an essential part of learning to do mathematics. - p. 30

By **attending to the precision** of their mathematical communications, whether spoken, written, or sketched, students can maximize the opportunities for others to understand their ideas. - p. 75

## Getting into Position

"Doing mathematics" for meaning-making requires that students and teachers reposition themselves: students as questioners and thinkers, and teachers as guides. Both teachers and students may be nervous when these ways of interacting are new. Teachers can help everyone move forward when they steadily support classroom expectations that students should respond to one another's ideas, that every voice should be heard, that all ideas count, that questions are good, and that "wrong" answers help everyone move forward. Over time, students develop experience and increased confidence as meaning-makers in mathematics.

Helping students become effective and collaborative sense-makers in mathematics means that teachers, like orchestra or choir conductors, set and maintain a productive tone and rhythm by calling upon different instruments or voices at just the right moment, signaling when each should come in, knowing which should be emphasized, and layering one with another to portray the full complexity of the music (or the math) being conducted. Before a discussion even begins, teachers' preplanning sets the stage. By selecting high-quality tasks that will give students opportunities to engage in discourse and use mathematical tools, and planning activities to foster interaction, teachers create a rich context for meaning-making.


**5 Practices for Orchestrating Productive Mathematics Discussions (Smith & Stein, 2011)**

Through careful preparation, teachers can anticipate the range of student approaches and then monitor them while circulating among student groups. They can then select and sequence specific approaches for the whole group to consider. In this way the teacher is able to use evidence of student thinking to build a bridge to the mathematical concepts at the heart of the lesson (Smith & Stein, 2011). It is student reasoning (carefully probed and amplified by teachers) that leads to deeper understanding.

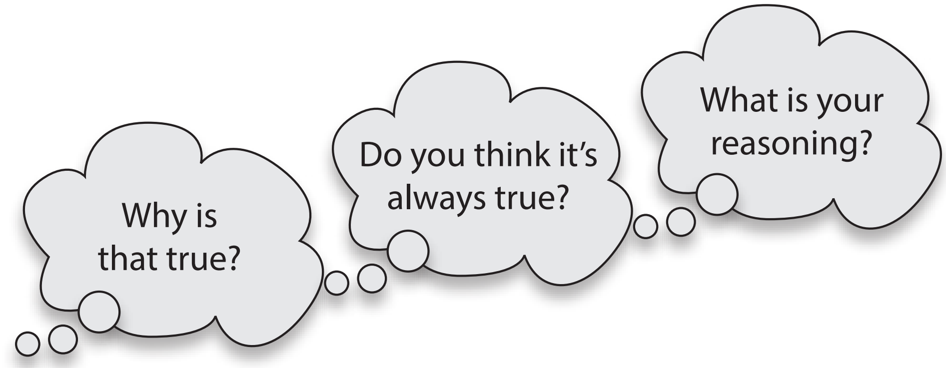
## Asking the Right Questions

**Questions that provoke productive struggle: big questions.** Helping students learn to do mathematics for meaning involves helping them develop what *Adding It Up* (NRC 2001) calls a *productive disposition:* the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that the steady effort in learning mathematics pays off, and to see

> The resources by Leinwand et al and by Smith and Stein, listed under Additional Resources, offer helpful guidance on choosing activities that promote deep reasoning.

oneself as an effective learner and doer of mathematics (p. 131). Reaching these big goals requires working with big ideas. If we want to help students reason their way through to the mathematical sense at the core of *something*, it follows that the something must be challenging enough to capture, hold, and be worthy of their attention. Simply stated, the task should connect to a big idea in mathematics, and must present the sort of puzzle for which there are multiple entry points and varied solution strategies.

**Questions that strengthen student reasoning.** Understanding the big idea in mathematics does not happen in one step. Experienced teachers have several sets of purposeful questions ready to help students defend their thinking, look for repeated reasoning, make generalizations, and prove those generalizations. Facilitating that progression of reasoning takes skill and practice. The **Productive Conversation Prompts** offer examples of how to press students for meaning and clarity, and how to support their efforts to understand. The **Classroom Supports** chart offers a broad array of supports that all students can use as they work to construct meaning together.




**Questions that orchestrate interaction.** It will be clear by now that productive discussion is a key component of sense-making in mathematics. Complex knowledge and skills are learned through social interaction as students share ideas, use one another as resources, and together construct new ways of understanding.

"A classroom is a community of learners. Communities are defined, in part, by how people relate to and interact with each other… It must be remembered that interacting is not optional: it is essential, because… communication is necessary for building understandings." Hiebert et al, 1997, p. 9

*Meaning is not stored language. Meaning is stored experience.*

Placing students into small groups where they are accountable to one another provides an "All brains to work!" opportunity. But students need instruction and support to leverage the power of shared thinking, since few students come to school knowing how to reason productively together. Some of the **Productive Conversation Prompts** mentioned above are designed to promote students' collaborative reasoning. Similarly, students need support in learning how to interact as they build ideas. Issues such as turn-taking, responding in connected, cohesive ways to previous comments, following a chain of reasoning, and challenging someone's idea without challenging the person are all important skills that can be learned and practiced. The

See the bulletin on group work by Lee, Cortada, & Grimm, listed under Additional Resources, for ways to support effective group work for English Learners.

**Productive Conversation Moves** show seven common interactions as students respond to one another's ideas, and offer models for these interactions across the range of English proficiency. The **Conversation Support Card** can be used by small groups of students as they work to build meaning together.

Chapter 8 in the book by Zwiers, O'Hara, and Pritchard, mentioned in the Additional Resources section, is an excellent source of advice and practical strategies for teaching these interaction skills.

### Thinking about English Learners: A few modifications to make

English Learners have more in common with their English-fluent peers than they have differences, but attending to those few differences can help culturally and linguistically diverse students become successful meaning-makers with their classmates.

Although they sometimes know two or three other languages, English Learners start at different points than their classmates when it comes to English. Those whose proficiency in English is just beginning to develop need ideas presented in multiple ways. Students whose English proficiency is more developed need fewer and different supports, and are more like their English-fluent peers; with their greater ability to express themselves, they too are learning the vocabulary and language patterns to express new and complex ideas, and to make more precise meanings in mathematics. Their interaction with English-fluent peers as they work out ideas collaboratively provides the impetus to learn new ways of using English to express ideas clearly. The **Productive Conversation Moves** and **Conversation Support Card** are all designed to assist English Learners at all levels of English proficiency to express their responses to ideas within their groups. The **Classroom Supports** list many ways teachers can support English Learners as they do the extra work they must do to create meaning in an English-speaking context, and many more suggestions are readily available. As always, it's important to think about the principles behind the strategies you use. Here are some that apply to our context:

1. Remember three key words: repeated, multiple, and deep. English Learners need **repeated** exposure to and **multiple** experiences with ideas in order to connect the ideas to the English used to express them. They also need information conveyed in **multiple** ways—voice, print, pictures, activities—so that language alone does not bear the full weight of meaning-making. And English Learners need **deep** experiences. Rather than present English Learners with new materials to process, it can help to work with the same materials they've just encountered, but to do it more deeply: answer new questions, produce something different, consider the ideas from a different vantage point. As mentioned earlier, meaning is not stored language; meaning is stored experience, and English Learners will build meaning through multiple related experiences.

2. Provide opportunities to discover and discuss the relationship between meaning and language. Questions such as: *How do you know that's what it's asking? Where does it say that? What makes you think the writer is uncertain? Where does it show that she is considering a counter-argument? What words tell you that? Which writer is more convincing and why?* can help students explore the connection between meaning and language.

3. Discuss the relationship between linguistic choices and the disciplinary purposes it serves. When pressing student to be more precise in their reasoning *(Is it always that way? What if…? How do you know that?),* take time to discuss why you are pushing them to be more precise, or more detailed, or more objective. This provides wonderful opportunities to convey the values of your discipline and to help build the habits of mind students will need.

**Student and teacher positioning.** Some English Learners, like some of their classmates, may come from families in which it is not appropriate for students or children to ask questions or challenge someone's reasoning. These family views may be supported by cultural patterns that view knowledge as a commodity in the hands of highly trained and highly educated "others" who are responsible for passing that knowledge on. Whatever its source, this view is worth exploring in the classroom. Explaining to students and families that the teacher's role is not to give answers, but to teach students to think, can shift perceptions. Similarly, if some students have been positioned by their classmates or families as either "poor thinkers" who can have no worthwhile contributions, or as "brains" who must have all the answers, patience and persistence in helping all students express their ideas and reason collaboratively can help shift those patterns.

## Meaning-Making in Mathematics

Mathematics emphasizes using valid, logical reasoning to determine whether a mathematical statement is true or false. Students may begin at the stage of *empirical reasoning,* giving examples to show that something is likely to be true. At the next stage, *informal reasoning,* students may be able to describe some patterns or general features that apply across examples. At the *formal reasoning* stage, students' reasoning is based not on examples, but on formal logic (Koestler et al, 2013, p. 30). Guiding students through these stages of reasoning takes skill and practice in knowing how to ask the right questions at the right time.

The **Productive Conversation Prompts** presented here provide guidance in facilitating discussions that deepen students' mathematical reasoning. Organized by instructional purpose, the chart offers guidelines for orchestrating discussions that focus student attention and valuable discussion time on the clarification and deepening of student reasoning.

| Orchestrating Purposeful Discussion: Productive Conversation Prompts to Develop Student Reasoning in Science<br>Adapted, in part, from the Tools for Ambitious Science Teaching website and Michael's & O'Connor (2012) | |
|---|---|
| **A. Help individual students clarify their thinking**<br>Giving students time to think before they answer questions, and then waiting again after they've voiced their initial comments, promotes deeper thinking and more elaborated and carefully considered responses.<br>• Use 10-20 seconds of wait time after questions and after responses.<br>• Use pair activities to help students clarify ideas before speaking to larger group.<br>• Write, draw/sketch or use materials to support student thinking in partners or individually.<br>Have students reflect on and explain their thinking, individually or in partners, using whatever combinations of language students wish to use: first language, combination of English with other languages, informal non-science language.<br>Ask students to identify and reflect on related experiences, discussions in their first languages, and school-related experiences/ideas that they can draw from. | **D. Help students listen carefully to and think about others' ideas**<br>"Who can rephrase or repeat?"<br>"How can you show (gesture, act out) their idea or identify the model or big idea that the student is using in his explanation?"<br>"How can you draw a picture/sketch of what they are saying?"<br>"How can you identify the evidence that they used in their explanation?"<br>"How can you indicate agreement or disagreement with their statement?"<br>"Whose idea/thinking is most different from your own?"<br>"How can you show or tell us a way their idea changed your questions or your thinking?" |

| | |
|---|---|
| **B. Make ideas and thinking public and available for discussion, analysis, and agreement or disagreement**<br>Press for additional info: ask for students' reasons or support for ideas<br>• Ask for example.<br>• Ask to fill out or extend explanation.<br>• Manage silence--extend wait time, after both questions & responses.<br>• Ensure clarity of ideas expressed.<br>• Clarify/repair how idea is expressed, without overriding student's ownership of idea; ask student for confirmation of your paraphrase.<br>• Re-voice to connect everyday expression to more precise academic language.<br>• Ask students to explain other classmates' thinking in whatever language or combination is accessible.<br>Prompt students to examine/seek out discrepancies of model or explanation.<br>Ask students to provide reasoning to support or revise models or explanations. | **E. Help students deepen their reasoning**<br>Ask for rationale.<br>Ask how students would test that idea.<br>Engage students in a phenomenon that is inconsistent with the explanation or model. Ask, "How can we revise the model/explanation to account for this new evidence?"<br>Compare two student-generated models or explanations and analyze how the class' collective thinking has changed based on the evidence collected. (This can also be done individually, looking over personal work.)<br>Collectively, sort the available evidence for consistency and inconsistency with model or explanation.<br>Ask, "What new questions occur to you now after the experience and discussion?"<br>Ask, "What do you need to know more about now?" |
| **C. Mark/emphasize a particular idea.**<br>"Rebroadcast" an idea through revoicing, always making sure that the ownership of the idea remains with the student who voiced it originally, always checking with that student to make sure you've expressed the idea as he or she intended.<br>"Rebroadcast" an idea by asking a student to paraphrase it. This gives the idea more "air time" and gives everyone a chance to hear it and think about it again.<br>Be explicit about putting some ideas on hold to focus attention on the idea you want to emphasize. | **F. Help students build meaning with others by applying their own thinking to others' ideas; prompt peer-to-peer talk**<br>Prompt for paraphrase and confirmation of approval of restatement.<br>Prompt students for frequent use of comprehension checks and clarifying questions.<br>Prompt for clear connections between student's response and previously stated idea.<br>Consistently model valuing others' ideas, experiences and perspectives.<br>Model valuing whatever language combinations students use to build meaning. |

# Meaning-Making in English

English Learners are important contributors to classroom meaning-making. They may not yet be fluent in academic English, but they can use their own creativity and the supports offered by teachers and classmates to reason with others.

## *No one is a native speaker of academic English.*

No student comes to school knowing what educators call *academic English.* The English used in school is a specialized version, one of many types of English that we develop to suit different situations, and is different from the English used in everyday conversations. (Just as there is an academic English, there is an academic version of French, Spanish, Arabic and other languages.) The language of mathematics is an even more particular version of that specialized academic English. Many words used in mathematics are also used in everyday language, but in mathematics, they have different and often more precise meanings: *difference, similarity, angle, point, factor, multiple.* Additionally, mathematics focuses on certain types of relationships between ideas and patterns of thinking that lead to specific ways of using language. Students learn these patterns as they need them, and they learn the language of mathematics by "doing mathematics."

English Learners may be able to express complex mathematical ideas in two or three languages, but have different starting points than their classmates when it comes to expressing those ideas in English. Their English is less proficient than that of their peers, but, like their classmates, they will learn through collaboration and experience the specific variety of English they will need to communicate their growing understanding of mathematics. The tools shown in this section are designed to help teachers support English Learners' use and development of English as they engage in meaning-making interactions. Often, teachers find these tools helpful for many of their students. After all, English Learners are not the only students working to develop precise, sophisticated language.

**Meaning-Making Tools**

- Productive Conversation Prompts
- Language Development Moves
- Productive Conversation Moves for Interacting with Ideas
- Conversation Support Card

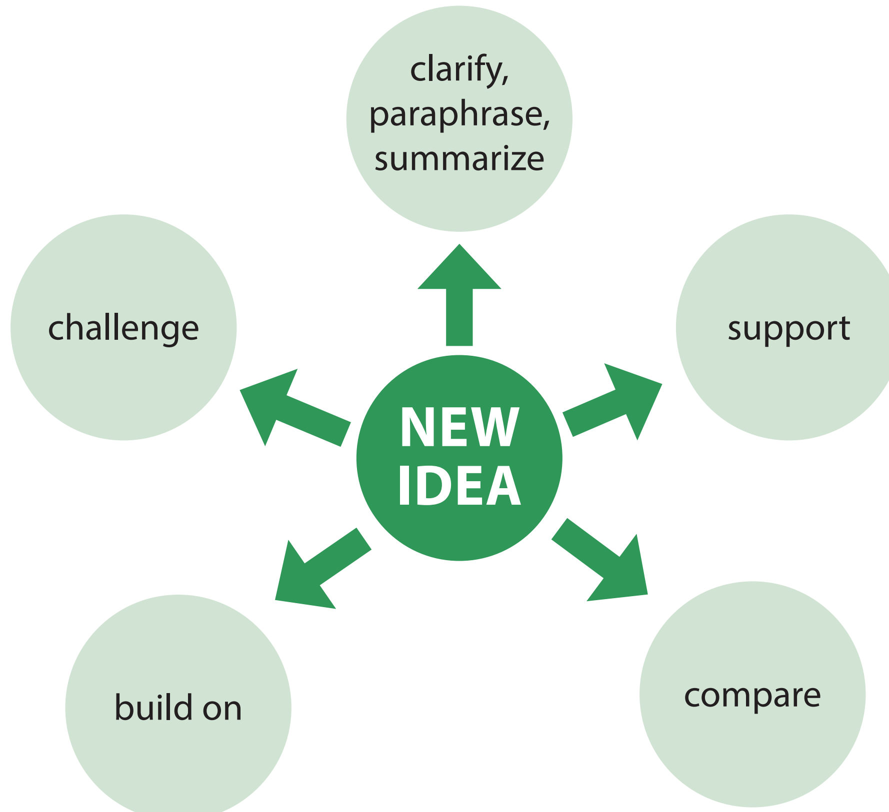
### Productive Conversation Moves: How English Learners Can Interact with Ideas

The first section of the chart below lists a few characteristics of students' capabilities in English at three broad levels of proficiency. Although some states categorize English proficiency in five or six levels, the three-level descriptors from which these characteristics are derived are based on and consistent with English proficiency descriptors used across the U.S. (Cook & MacDonald, 2014), making it fairly simple to translate across different systems of classification. Whatever system is used to describe English Learners' growing capacity with English, it's important to remember that English development doesn't always follow expected patterns. Words that seem like advanced vocabulary in English may be similar enough to words in students' other languages that they are learned very early. Similarly, aspects of English that are learned fairly quickly by some may be more challenging to others whose first language may just work very differently than English. It is not critical to have a highly accurate rating of students' English proficiency. What's more important is understanding some aspects of English development and how to provide a range of supports to help English Learners contribute their ideas to classroom discussions.

The remainder of the chart is designed for both teachers and students to use. We developed a list of seven types of responses students would make to interact with one another's ideas:

- Tell and support one's own idea
- Ask for clarification
- Restate or paraphrase an idea
- Summarize ideas
- Support someone's idea
- Build on someone's idea
- Challenge someone's idea

For each of these seven response types, the **Productive Conversation Moves** below show how English Learners can discuss and develop ideas with others at all three levels of English proficiency. Teachers can use these to provide written or oral prompts for student responses. English Learners can use them to help formulate responses. All students can use them as they learn and practice the interaction patterns of academic conversations.



| What students can produce in English at three levels of proficiency | | |
|---|---|---|
| **Low English proficiency** | **Intermediate English proficiency** | **High English proficiency** |
| Short, simple expressions with common vocabulary. Some simple connectors (because, so, then) may be used. | More involved sentences that contain more than one idea, with a broader range of logical connectors used to create additional types of logical relationships between ideas. | More concise sentences with ideas embedded within other ideas. Personal pronouns removed to maintain coherent and impersonal focus on concept rather than on actor. Personal opinion expressed indirectly. |
| **Productive Conversation Moves: How English Learners can use English to …** | | |
| **Tell and explain a new idea** | **Effective explanations in math**<br>• Spell out the logic/reasoning involved<br>• Are supported by examples<br>• Are supported by reference to previously learned principles | |
| *I think…My reason is…*<br>*My idea is…*<br>*I think Lupita is right. My reason is I got the same thing.* | *We should try… because*<br>*Since the pattern is the same, I think we could…*<br>*We know that Lupita is wrong because if you…* | *The obvious solution is …*<br>*We know that Lupita's strategy doesn't work because we drew an array, and we noticed that Lupita only found two of the partial products instead of finding all four.* |
| **Clarify** | **Effective questions in math**<br>• Ask specifically for restatement, or clarification, or additional explanation | |
| *Say again, please. (Seeking repetition)*<br>*What is … ? (Seeking more information)*<br>*Why did you draw that? (Seeking rationale)* | *What did you mean when you said that… (Seeking more information)*<br>*Can you tell me why you think that … ? (Seeking rationale)*<br>*What did you mean when you said that Lupita didn't find all of the partial products?* | *It seems the suggestion is…. Is that correct?*<br>*What is the rationale for that change?*<br>*You seem to be arguing that Max should have written a different equation to explain why Lupita's strategy doesn't work. Is that correct?* |
| **Restate or summarize an idea** | **Effective restatements/summaries**<br>• Need not be grammatically correct to be effective<br>• Represent underlying math constructs accurately<br>• Are checked for comprehension, and increase in precision until accurate representation is attained | |
| *He said "…" (may attempt direct quotation; not likely to produce full details)*<br>*It means …*<br>*It is the weight. (statement of main point, not embedded into another framing sentence.)*<br>*Juan said he thinks Lupita is wrong.* | *In other words, …*<br>*What Sara means is that there are 13 rows of 18 squares.*<br>*We have 3 different ideas about why this could be happening, and they all relate to…* | *The drawing was changed to show…*<br>*The suggestion was made that …*<br>*Every idea expressed so far has centered around…*<br>*The problem encountered by every group has been that…*<br>*Kaamya meant that the array that Fudi drew should be divided into four sections, not two.* |

| | | |
|---|---|---|
| **Compare ideas** | **Effective comparisons in math**<br>• Compare solutions<br>• Compare underlying principles | • Compare strategies<br>• Compare situations or contexts |
| *It always works* | *So far, everyone's suggestions are about…*<br>*This works with positive and negative numbers.* | *Our strategy is most like group A's because….*<br>*That explanation is a better fit, since it accounts for….* |
| **Support someone's idea** | **Support for an idea needs to be based on**<br>• Additional examples of why strategy or reasoning is correct<br>• Reference to previously learned mathematical principle | |
| *Good idea because…*<br>*I agree with Yer because….*<br>*I like what Dariela said. I drew the same picture.* | *Remember, it said in our book that…*<br>*I'm sure that would work since...*<br>*That idea makes sense because last week, we learned…*<br>*Sara's idea makes sense because her array shows that there are way more than 124 squares.* | *That explanation is a good one, since it accounts for….*<br>*The advantage of the proposed change is…*<br>*Jonathan's explanation is the most powerful because he used both equations and an array to demonstrate why Lupita's strategy didn't work.* |
| **Build on someone's idea** | **Effective building on ideas is based on**<br>• Explaining how it relates to or demonstrates a math principle<br>• Giving additional examples of why it works | |
| *Let's try that.*<br>*We can draw a picture too.* | *What if we try it this way, instead of…?*<br>*Let's change the model to show that.*<br>*Yer's idea made me wonder about…*<br>*After we draw the array, we could write some equations that show how we figured out the answer.* | *The obvious next move would be to…*<br>*Based on what Jonathan said, I think it would probably be helpful to build the array with base 10 blocks so that everyone else can see clearly which partial products Lupita was missing.* |
| **Question or challenge someone's idea** | **Challenges to an idea should make reference to**<br>• Mathematical principle that contradicts idea<br>• Exploring related constraints | • Example that disproves a statement |
| *I think that's wrong because…*<br>*It won't work because...*<br>*What is your evidence?* | *But your explanation will not account for…*<br>*Did you think about…?*<br>*How does the evidence support your idea that______* | *Unfortunately, that explanation doesn't fit every situation.*<br>*The situation you've neglected to account for is…*<br>*Had you considered…?*<br>*Do you think that there might be a more efficient way that you could multiply 13 and 18?* |

Note: Sentences described here for the high English proficient level may not occur in the early stages of students' thinking or in a small group context, even for high English proficient students. This more formal expression occurs more naturally in and is better suited to presentations or writing. Nonetheless, students need instruction to develop this high-level English, with strategies such as those described in the **Language Development Moves**.

# Classroom Supports for Meaning-Making in Math

Here is an extensive array of strategies to support English Learners' sense-making in mathematics. Fewer and different supports are needed as students gain proficiency in English, but note that even students with high English proficiency benefit from support and instruction.

| Classroom Supports for Meaning-making in Mathematics | | |
|---|---|---|
| **Low English Proficiency** | **Intermediate English Proficiency** | **High English Proficiency** |
| **Home discussion questions** to involve families in discussions of the mathematics being investigated.<br>**List** of graphically-supported key words and their meanings.<br>**Student-generated lists** of terms and ideas in English and other relevant languages.<br>**Sketches, charts, and other visual supports** at hand to support telling of rationale, with teacher assisting as needed to voice as students points, or to re-voice to clarify meaning (Note: focus on meaning, not on correctness) or the means for students to create their own mathematical representation by drawing to show their thinking with words, pictures, and symbols, or using manipulatives to show what they mean, or displaying their paper on a document camera.<br>**Sufficient wait time** to allow students to formulate ideas in English.<br>**Purposeful grouping:** pair work to help students put ideas into words, progressing to small heterogeneous group work for low-pressure language formulation and peer assistance.<br>**Sentence frames** that model the use of phrases or simple sentences to accomplish the interaction. *(See sentence frames specific to each interaction.)*<br>**Visual portrayal of evidence** presented so far.<br>**Continuously revised conceptual web** labeled in relevant languages, with sketches or graphic supports.<br>Charts and reminders to **support periodic tracking of group's collaborative sense-making.** *"Let's stop for a minute. I want everyone to focus on the conversation that Yritzy and Alan just had. What did you notice?"* | **Home discussion questions** to involve families in discussions of the mathematics being investigated.<br>**List** of technical content-area words and their meanings.<br>**Chart** of language cues to various response types (support, challenge, etc.) to help students comprehend the discourse function of statements they hear.<br>**Sketches, charts, and other visual supports** at hand for student to point to or refer to as needed or the means for students to create their own (i.e., mathematical representation).<br>**Sufficient wait time** to allow students to formulate ideas in English.<br>**Purposeful grouping:** small heterogeneous group work for low-pressure idea formulation and language experimentation.<br>**Sentence frames** that model more compact expression by combining more than one idea per sentence to accomplish the discourse function. (See sentence frames specific to each discourse function.)<br>Charts and reminders to **support periodic tracking of group's collaborative sense-making.** *"Let's stop for a minute. I want everyone to focus on the conversation that Yritzy and Alan just had. What did you notice?"*<br>**Evidence collected** so far in central location with quick sketches of the evidence.<br>**Continuously revised conceptual webs.** | Supplies for students to draw or make their own **mathematical representation.**<br>**Sentence frames** that model a more objective or authoritative expression of an idea to accomplish the interaction. (See sentence frames specific to each interaction.)<br>Charts and reminders to s**upport periodic tracking of group's collaborative sense-making.** *"Let's stop for a minute. I want everyone to focus on the conversation that Yritzy and Alan just had. What did you notice?"*<br>**Guided examination** of rich examples of student generated talk (written down), student writing, and professional science writing to examine the linguistic choices made and their effects. |

This broad array of supports can be used and adapted in many ways. Some teachers vary the supports provided to different working groups of students. Others choose to make all supports available to everyone, trusting that English Learners—and others—will gravitate toward those that they need.

### Conversation Support Card

When students collaborate to create meaning, much of the interaction occurs in small group discussions, and teacher support is not always available. When students have begun to learn how to discuss ideas together, they can use a **Conversation Support Card** like the one shown in this guide. Cards like these, developed to suit specific activities, can be placed at each table as tools to help student groups keep their conversations moving along productively. Students can be involved in the creation of cards like these, brainstorming many ways to word various responses.

The book by Zwiers, O'Hara, & Pritchard, listed in Additional Resources, has excellent examples of similar tools.

### Language Development Moves

Like their classmates, English Learners need instruction to support their English development. When students work together to express important ideas clearly, the interactive context helps students push themselves and one another in the direction of more effective expression of ideas. Working with complex ideas pushes the development of complex language. Students whose English proficiency has moved beyond the basic beginning stage can benefit from instruction and modeling in additional ways to make meaning in English. Indeed, if the ideas they're working with are engaging, they will be eager to find ways to communicate more easily with their English-fluent classmates.

This section provides an overview of some language development strategies that can be integrated into mathematics lessons. Instruction should always be tied to an experience students have had and to their perceived need for additional ways to clarify ideas. It should use real language generated in class. It is also important—especially when students are introduced to the highest level of English proficiency, in which they learn to adopt the voice of a mathematician—to discuss the advantages and disadvantages of these new ways of expressing ideas.

Mathematical language is very precise, which sometimes causes it to be very formal, as well. But students can be very precise without using formal language. Having students examine several samples of writing in mathematics and discuss the different styles of English used can lead to fascinating discussions of how language is shaped by the values of mathematics and by the different contexts in which mathematicians operate. Students can enjoy debating the advantages and disadvantages of using different styles of English to move across these multiple contexts.

| Moving toward intermediate English proficiency | Moving toward high English proficiency |
| --- | --- |
| • Describing complex things precisely, or building long "things"<br>• Compacting ideas | • Adjusting the message to the context |

# Helping Students Move from Low to Intermediate English Proficiency

**Describing complex things precisely, or building long "things" in English.** One of the first things students may notice when they read mathematics texts or problems is that the sentences are very long. Students can learn relatively early in their English development that English can create very long "things" with words. Consider this sentence:

*Use a protractor to measure the angle formed by the back leg of the chair and its shadow on the ground in all three pictures.*

What is the "thing" that is to be measured? Not simply *the angle,* but the *angle formed by the back leg of the chair and its shadow on the ground,* a long phrase that identifies one specific angle. Mathematics is specific, and details matter. Mathematical language is also efficient, and putting all the important details together in one long phrase is faster that stringing the details out across several sentences. It makes the expression more precise.

Long linguistic things (long noun phrases) like the phrase above have been described as the powerhouse of meaning in English and are one of the first things that students change as they move into using academic English. Spending a bit of time showing students this pattern can help them both comprehend and use this strategy for describing specific things or concepts very precisely. Simple and quick activities such as asking students to embed the details from Sentences 2 and 3 into Sentence 1 (all below) can be fun and helpful.

*Measure this angle. One side is formed by the back leg of the chair. The other side is formed by the shadow on the ground.*

**Compacting ideas.** Students can make accurate and important meanings with conversational style English, which usually has just one idea or event in each sentence. The more information the students need to convey, however, the greater the number of small sentences they must string together. Consider the following sentences:

*We are going to do an experiment with a bag of 11 candies.*
*5 of them are cherry.*
*6 of them are lemon.*

*We are going to do an experiment with a bag of 11 candies, 5 of which are cherry and 6 of which are lemon.*

The first set of sentences is most likely how this situation would be described in conversation, and in elementary grade lessons. But beyond those early grades, the precision and efficiency of mathematical language compacts those three sentences into one. The ideas in the second and third sentence are embedded into the main sentence. This is a very common pattern in mathematics, and giving students practice in deconstructing and reconstructing sentences like this will help them decode them more quickly when they encounter them.

## Helping Students Move from Intermediate to High English Proficiency

**Adjusting the message to the context.** As English Learners get more comfortable creating long “chunks” of language, they can insert them into different places in a main sentence or combine them in different orders for a range of effects. In other words, they can create more complex sentences. But high proficiency in English is more than complex language. High proficiency involves knowing how to tailor messages to specific circumstances. Proficient users of English consider questions such as:

- *Can my listener see the object I'm referring to?*
- *Will people be reading this or listening to it?*

Considering these questions is part of taking on the value and voice of a mathematician by conveying meaning as precisely as possible. Precision is a dialogic concept, which simply means that precision is “in the eye or ear” of the reader or listener. As students develop greater experience with English, they are more able to shape their messages to the differing needs of readers and listeners across different contexts. They become better meaning-makers in more contexts. Listeners, for example, do better when ideas are not packed too densely, when there are time and space between major chunks of meaning so that they can be processed one at a time. This string of sentences

*Measure this angle. One side is formed by the back leg of the chair. The other side is formed by the shadow on the ground.*

is much simpler for a listener to comprehend than its more compacted counterpart. Readers, on the other hand, can read back over a dense sentence to extract the details, and appreciate having the important information clumped together.

A skilled user of English is able to respond to the presence or absence of a shared context with the receiver of the message. Consider the following set of sentences:

A. *Look at this line. See how long it is? It's way out of proportion to this other one.*
B. *That bottom line looks too long. It's out of proportion to the other one.*
C. *The line parallel to the base of the triangle is disproportionately long when compared to the line perpendicular to it.*

When speaker and listener are in the same room, looking at the same drawing, it makes perfect sense to use that situation as part of the meaning-making. Supplemented by visual support, sentences A and B would lend themselves to precise meaning. It would be silly not to use the opportunity to simply point to a line, rather than to give a linguistically precise (and lengthy) description. But when there is no shared context, and language alone must bear the weight of meaning-making (or language in combination with a chart or drawing that must be described and referenced), then the long phrases (long “things”) referred to above become increasingly important.

Analyzing and discussing sets of sentences like these can help students appreciate the needs of those with whom they're making meaning. English Learners moving to the high English proficiency level are, like their classmates, learning the specific language of mathematics. They are learning to adopt a mathematical voice and to portray, through language, the precision and specificity critical to mathematical understanding. To gain important understanding and practice with this, English Learners and their classmates can compare different kinds of texts or oral presentations to identify the differences in the way English is used. Discussing the effects these linguistic choices produce helps students become more aware of the variety of messages and impressions they can choose to construct as they create meaning together in mathematics.

## Conclusion

Like the English Learners and their English-fluent peers in our classrooms, teachers learn by doing, and by doing things together. We hope that you will experiment with using these tools in whatever ways fit into your classrooms, that you'll discuss your ideas with your colleagues, and that you'll refine these tools together as you go. As you interact in your own spiral of meaning-making in mathematics education, we're confident that the interactions you support among your students will deepen their knowledge and help them make meaning of the world we all share.

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# Lesson Scenario

This third grade mathematics lesson has been adapted from *Implementing the Common Core State Standards through Mathematical Problem Solving, Grades 3-5* (Foote et al., 2014) as an example of what an interactive, meaning-focused mathematics lesson looks like. It's also an example of how teachers and students can use supports to reason effectively together, even with developing English. The lesson uses many of the supports and strategies in this guide—many more than can be shown neatly—but a few are marked in bold type to demonstrate how easily and naturally they fit into this mathematics classroom.

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### Lupita's solution

Ms. Morales teaches a class of third and fourth graders in a vibrant, diverse elementary school. Her class consists of 24 students, 16 of whom are English Learners (ELs). For nine of the ELs, Spanish is their first language. The other ELs include speakers of Mandarin, Tagalog, Russian, and Hindi. The ELs have been learning English for varying lengths of time. Some have been learning English since preschool. A few have only been learning English for the past year or two.

Ms. Morales and her students are in the midst of a unit on multiplication and division, in which they are exploring various informal strategies for solving problems involving multi-digit numbers. Students are often asked to solve problems using the strategy that makes the most sense to them. Some students typically solve problems by modeling all quantities in the problem and counting to find the answer. Others use skip counting, or repeated addition or subtraction. Students have been working on their multiplication facts, and apply known facts to help them solve problems. Students are starting to use strategies based on place value, such as partial products and partial quotients strategies. Students are becoming familiar with several models for multiplication, including equal groups models, linear models, and array models.

For today's lesson, Ms. Morales **wants students to explore an error that children often make** when they are learning to use a partial products strategy. For this reason, she plans to pose the following problem:

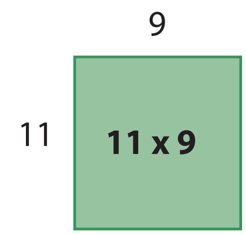
> *Lupita solved 13 × 18 by multiplying 10 × 10 and 3 × 8. Then she added 100 and 24 to get 124.* **What do you think** *of Lupita's strategy? Does it work? If you think it does,* **can you prove it?** *If you think it doesn't work,* **how would you convince** *Lupita?* **Why** *doesn't it work? How would you help Lupita?*

As she plans for the lesson, Ms. Morales thinks about various goals she has for her students. She **wants to deepen students' understanding** of the partial products strategy—an efficient and generalizable strategy for multiplication, and one that can help students make sense of the standard algorithm. She hopes to build on students' emerging understanding of the distributive property. She wants to see what models students use to think about this problem, and plans to connect their thinking to an array model—a powerful model for making sense of the partial products strategy. Additionally, she **wants to help students engage in powerful mathematical practices,** including constructing viable arguments and critiquing the reasoning of others, attending to precision, and looking for and expressing regularity in repeated reasoning. Ms. Morales also wants to support all of her students as they use English to learn math, and math to learn English.

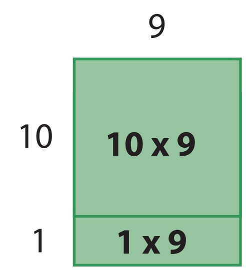
*Note how the teacher has selected a challenging task and has posed questions with high cognitive demand.*

Before she poses the task, Ms. Morales **considers what her students already know,** and her knowledge of how students typically think about problems such as this one. She anticipates that some students will initially agree with Lupita, since it is common for students to think that they can multiply multi-digit numbers by first multiplying the tens, and then multiplying the ones. After all, that is how they solve addition and subtraction problems. She anticipates that some students will solve the problem in a way that makes sense to them and will determine that Lupita's strategy doesn't work because they don't agree with her answer. She anticipates that determining the specific error in Lupita's strategy will be challenging for many students.

As Ms. Morales launches this task, she has gathered her students together on the rug in front of the class whiteboard as she generally does for **whole-class discussions.** Because Ms. Morales wants to connect students' thinking on this task to an array model, she first poses an easier problem to which most students already know the answer: 11 × 9. Students call out the answer, and she writes it on the board. She asks students how they might represent this problem with an array, and students tell her that she could have 11 rows of 9 squares, and she draws this. She then reminds them that they don't always have to draw all of the squares, and draws an "open" array.




She asks students how they might use partial products to solve this problem. A student responds that they could do 10 × 9 and 1 × 9. She asks another student how they might show this on the array and a student comes to the board and draws a line on the array. She erases the 11 on the side, and replaces it with 10 and 1, and then writes the corresponding multiplication expressions.




Ms. Morales thanks the student and tells the class that an array model might help them with today's task, but she leaves it open to the students whether or not to use this model. She tells the students that today, they are going to think about the strategy used by a student named Lupita, and she **writes the task** described above on the whiteboard. She asks a student to **read the task out loud** for the class. Then, she tells students to turn to a partner and explain to them what the task is asking them to figure out. After this **turn-and-talk,** Ms. Morales **calls on a few students to share in their own words** what they are being asked to do.

*Notice the teacher's use of multiple modalities (speech and writing) to present information, and the way she checks her students' comprehension.*

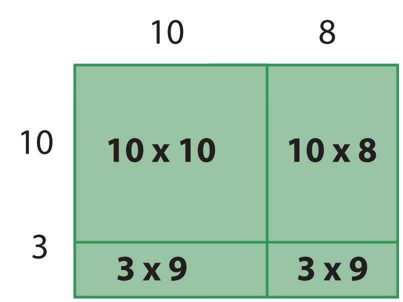
Ms. Morales next explains how the class will explore this problem. She tells them that first, **each student will spend about five minutes thinking about the problem quietly on his or her own.** They might get started on the problem, or they might think about questions they have if they are not sure how to get started. Then, students will have about twenty minutes to **work with a small group** on this task. She reminds students that there are **posters on the wall with conversation moves** that they might use during their group work. She tells students that they can **use whatever math tools they might need,** including square tiles, base ten blocks, graph paper, blank paper, and rulers, all of which are on shelves students can access. Finally, Ms. Morales sends students back to their tables to get started.

During the five minutes allotted to individual think time, Ms. Morales has **brief, quiet conversations with a couple of students** who are having a hard time engaging with the problem. When five minutes have passed, Ms. Morales tells students that it's time to work with their small groups. **As students work in their groups, Ms. Morales circulates from table to table** having brief conversations with each group. She is **monitoring the groups' thinking, and starting to make decisions about which groups will share** during the group discussions at the end of the lesson. As she talks with groups, she asks questions which give her insight into students' mathematical understanding. She helps facilitate the groups' work and **supports students' language development by modeling** how they might ask a question to a group member, or indicate respectfully that they don't agree with someone. She sometimes revoices what a student says using more precise mathematical vocabulary. She asks questions that attend to her goals for the lessons, too—**pressing students to critique** Lupita's reasoning, **and to be specific** about why they agree or disagree with her strategy.

*Note the teacher's effective use of varied participation structures, and the resources she has placed around the room. Notice her use of Teacher Conversation Prompts and strategies such as revoicing and pressing for meaning and precision.*

During the last few minutes of group work, Ms. Morales **makes final decisions about which group will share, and in what order.** She talks with each group that will share during the discussion so that they are prepared to do so. She tells them that each group member should be ready to share, as she might ask any one of them to start. She reassures them that all group members will be able to help the person she asks to share.

Ms. Morales regathers the class on the carpet in front of the whiteboard. She uses a document camera and data projector to display the work of the groups that are sharing. The first group she asks to share at first thought that Lupita's strategy was correct, but at Ms. Morales' urging, they solved the problem a different way and got a different answer. She asks questions which help the students describe the process they went through over the course of their work together. The next group initially thought that Lupita's strategy had to be wrong because 10 × 18 is 180 and 13 × 18 has to be more than that because there are more groups of 18. So, they knew that 124 couldn't be the right answer. They explain that they thought about the problem as 10 × 18 and 3 × 18 and saw how this was different from 10 × 10 and 3 × 8.




Ms. Morales asks a student in the audience to **explain in her own words why this group disagreed** with Lupita. She asks another student how they might use this information to help Lupita. The next group explains how they used graph paper to draw an array that had 13 rows and 18 columns. They divided the array into four areas by splitting 13 into 10 and 3, and 18 into 10 and 8. The wrote the partial products in each section, and explained that that helped them see that Lupita hadn't done 10 × 8 or 3 × 10 and that was why her strategy didn't work. Ms. Morales **connects this group's work to the previous group's,** asking whether this picture could also show 10 × 18 and 3 × 18. Students agree that it could by eliminating the vertical line dividing 18 into 10 and 8. Finally, a fourth group shares briefly, explaining that they thought about the problem in much the same way as the previous group, but they just drew an open array, without the graph paper showing each square in the array.

To conclude the discussion, Ms. Morales **asks students to talk with a partner about why** Lupita's strategy didn't work. After students talk briefly, she **asks several students to briefly share** what they discussed with their partners. Then, Ms. Morales asks the students to talk with a partner about what might help Lupita avoid the same mistake in the future. Again, Ms. Morales accepts several responses, and highlights the idea that the array model is one way to make sure you remember all the partial products.

*Note the teacher's sequencing of student presentations, and her attention to helping students connect ideas. She has gone through all five of the steps recommended by Smith & Stein (2011), while integrating the Conversation Prompts to keep her students interacting with one another.*

*Throughout the lesson, Ms. Morales' focus on the meaning in the mathematics and her attention to helping students make meaning in English has provided a powerful learning experience for all of her students.*

## Math Conversation Support Card adapted from Zwiers, O'Hara, & Pritchard (2014)

# RESPONDING TO IDEAS

### Clarify, Paraphrase, Summarize

**Question Starters**

What do you mean by…?

Will you say that again for me?

But, what is the main point?

**Response Starters**

Another way to say that is…

What she means is that there are 13 rows of 8 squares.

You seem to be arguing that Max should use a different equation to...

### Support

**Question Starters**

What are some examples?

Can we try that?

Do you see the link to…?

Have we seen that before?

**Response Starters**

An example is…

I'm sure that would work, because…

That makes sense, because…

Jon used both equations and a graph to show…

### Build

**Question Starters**

How could that connect to…?

Would it work if we…?

Where could we use that idea…?

How can you add to that idea?

**Response Starters**

Let's try that.

The pattern is the same, so we could…

If we change to an open array, it would work better because…

After we draw an array, let's write equations that show…

### Question or Challenge

**Question Starters**

Did you think about…?

But, would it only work for…?

What is your reasoning?

Why do you think that?

**Response Starters**

I don't agree, because…

That ignores evidence of…

I see this very differently.

It doesn't work because when we drew an array, we noticed that…

## Additional Resources

The following books, articles, and websites offer excellent resources. Most are specifically related to working with English Learners. Some, such as the websites listed below, are about the teaching practices and discourse practices that help create meaning; these do also address meaning-making with new learners of English.

Council of Chief State School Officers. (2012). Framework for English Language Proficiency Development Standards corresponding to the Common Core State Standards and the Next Generation Science Standards. Washington, DC: CCSSO.

- Analysis of the language demands for English Learners
- Articulates the relationship between Science and Engineering Practices, analytic tasks, and the language needed for their accomplishment

Lee, N., Cortada, J., & Grimm, L. (2013). *WIDA Focus On: Group Work for Content Learning.* Madison, WI: WIDA Consortium.

- Excellent suggestions for using group work effectively with English Learners

Chapin, S., O'Connor, C., & Anderson, N. (2003). *Classroom Discussions: Using Math Talk to Help Students Learn.* Sausalito, CA: Math Solutions Publications.

- Some of the early work on mathematical discourse; many good suggestions for classroom teachers

Smith. M. & Stein, M. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions.* Reston, VA: NCTM.

- Brief and wonderful resource on teaching for meaning in mathematics

Zwiers, J. (2008). *Building Academic Language: Essential Practices for Content Classrooms.* San Francisco, CA: Jossey-Bass.

- Excellent and practical strategies for focusing on academic language

Zwiers, J., O'Hara, S., & Pritchard, R. (2014). *Common Core Standards in Diverse Classrooms: Essential Practices for Developing Academic Language and Disciplinary Literacy.* Portland, Maine: Stenhouse.

- Many strategies to strengthen students' ability to develop and sustain academic conversations
- Many additional resources on the related website: http://aldnetwork.org/core-practice/resources-fostering-academic-interaction

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Chapin, S., O'Connor, C., & Anderson, N. (2003). *Classroom Discussions: Using Math Talk to Help Students Learn.* Sausalito, CA: Math Solutions Publications.

Common Core State Standards Initiative (CCSSI). *Common Core State Standards for Mathematics.* Washington, D.C.: National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010. http://www.corestandards.org/

Cook, H.G., & MacDonald, R. (2014). *Reference Performance Level Descriptors: Outcome of a National Working Session on Defining an "English Proficient" Performance Standard.* Washington, D.C.: Council of Chief State School Officers, http://www.ccsso.org/Resources/Publications/Reference_Performance_Level_Descriptors.html.

Foote, M., Earnest, D., Mukhopadhyay, S., & Curcio, F. (2014). *Implementing the Common Core State Standards through Mathematical Problem Solving, Grades 3-5.* Reston, VA: NCTM.

Hiebert, J. (1997). *Making Sense: Teaching and Learning Mathematics with Understanding.* Portsmouth, NH: Heinemann.

Koestler, C., Felton, M., Bieda, K., & Otten, S. (2013). *Connecting the NCTM Process Standards and the CCSSM Practices.* Reston, VA: NCTM.

Lee, N., Cortada, J., & Grimm, L. (2013). *WIDA Focus On: Group Work for Content Learning.* Madison, WI: WIDA Consortium.

Leinwand, S., Brahier, D., & Huinker, D. (2014). *Principles to Action: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

National Research Council (NRC). *Adding It Up: Helping Children Learn Mathematics.* Kilpatrick, J., Swafford, J., & Findell, B. Washington, D.C.: National Academic Press.

Smith. M. & Stein, M. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions.* Reston, VA: NCTM.

Zwiers, J., O'Hara, S., & Pritchard, R. (2014). Common *Core Standards in Diverse Classrooms: Essential Practices for Developing Academic Language and Disciplinary Literacy.* Portland, Maine: Stenhouse Publishers.

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